

Non-commutative phase space and its space-time symmetry

Kang Li* and Sayipjamal Dulat†

*Department of Physics, Hangzhou Normal University, Hangzhou, 310036, China and
School of Physics Science and Technology, Xinjiang University, Urumqi, 830046, China
The Abdus Salam International Center for Theoretical Physics, Trieste, Italy*

(Dated: February 1, 2008)

First a description of 2+1 dimensional non-commutative(NC) phase space is presented, where the deformation of the planck constant is given. We find that in this new formulation, generalized Bopp's shift has a symmetric representation and one can easily and straightforwardly define the star product on NC phase space. Then we define non-commutative Lorentz transformations both on NC space and NC phase space. We also discuss the Poincare symmetry. Finally we point out that our NC phase space formulation and the NC Lorentz transformations can be applicable to any even dimensional NC space and NC phase space.

PACS numbers: 02.40.Gh, 11.10.Nx, 11.30.Cp

I. INTRODUCTION

In recent years, there has been an increasing interest in the study of physics on non-commutative space, because the effects of the space non-commutativity may become significant in the extreme situation such as at the string scale or at the TeV and even higher energy level. There are many papers devoted to the study of various aspects of the quantum field theory and quantum mechanics on NC space, where space-space is non-commuting, but the momentum-momentum is commuting, or on NC phase space, where both space-space and momentum-momentum are non-commuting. For references see [1]-[5]. Although quantum theories on NC space and NC phase space have been extensively studied in the literature, the description of NC phase space is far from complete, for example, it is not easy to define the star product on NC phase space in the formulations of Refs.[6] and [7]. Another important issue we want to discuss is the space-time symmetries of the NC space and NC phase space. As we know, the Lorentz symmetry plays a central role in any realistic quantum field theory. There are different approaches in the formulations of Lorentz and Poincare symmetries on NC space, for references see [8]-[10]. Ref.[8] studied Lorentz transformation on NC space and claimed that the NC gauge theories are invariant under the NC Lorentz transformations. Because of the singularity of matrix θ_{ij} the NC Lorentz transformation in Ref.[8] may not be applicable for 3+1 dimensional NC space. Also there is no discussions about the Lorentz transformation on NC phase space in the literature.

In this paper, we will give a description of NC phase space on 2 + 1 dimensions, where the star product can be easily defined. On 2 + 1 dimensional space-time, we extend the results in Ref.[8] to the NC phase space, and we find that the new formulation is applicable to any

even space dimensions.

This paper is organized as follows: in Section 2, we present a description of NC phase space on 2 + 1 dimensions. In Section 3, we discuss NC Lorentz and Poincare transformations both on NC space and NC phase space. Conclusion remarks are given in the last section.

II. THE DESCRIPTION OF NC PHASE SPACE

On NC space, the NC algebra is

$$[\hat{x}_\mu, \hat{x}_\nu] = i\theta_{\mu\nu}, \quad [\hat{x}_\mu, \hat{p}_\nu] = i\hbar\delta_{\mu\nu}, \quad [\hat{p}_\mu, \hat{p}_\nu] = 0. \quad (1)$$

where the Greek indices μ and ν run from 0 to 2. In order not to spoil unitarity [11, 12] and causality[13], one sets $\theta^{0\mu} = 0$; in two dimensions the constant anti-symmetric matrix element θ_{ij} can be written as $\theta_{ij} = \epsilon_{ij}\theta$, and θ related to the space-space non-commutativity.

Non-commutative field theories are constructed from commutative field theories by replacing, in the action, the usual multiplication product of fields with the star product of fields. The star product between two fields is defined as

$$\begin{aligned} (f * g)(x) &= f(x) e^{\frac{i}{2} \overleftarrow{\partial}_\mu \theta^{\mu\nu} \overrightarrow{\partial}_\nu} g(x) \\ &= f(x)g(x) + \frac{i}{2} \theta^{\mu\nu} \partial_\mu f \partial_\nu g + \mathcal{O}(\theta^2). \end{aligned} \quad (2)$$

This star product can be replaced by a shift which is called Bopp's shift

$$\hat{x}_\mu = x_\mu - \frac{1}{2} \theta_{\mu\nu} p^\nu, \quad \hat{p}_\mu = p_\mu. \quad (3)$$

where x_μ and p_ν are coordinates and momenta on commuting space-time. After applying this shift, the effect caused by space-space non-commutativity can be calculated in the commuting space.

The Bose-Einstein statistics on non-commutative quantum mechanics requires both space-space and momentum-momentum non-commutativity [7]. In the

*Electronic address: kangli@hznu.edu.cn

†Electronic address: dulat98@yahoo.com

following we present our formulation to NC phase space. On NC phase space, we set the NC algebra as

$$[\hat{x}_\mu, \hat{x}_\nu] = i\theta_{\mu\nu}, \quad [\hat{x}_\mu, \hat{p}_\mu] = i\tilde{\hbar}\delta_{\mu\nu}, \quad [\hat{p}_\mu, \hat{p}_\nu] = i\bar{\theta}_{\mu\nu}. \quad (4)$$

On NC phase space we set also $\bar{\theta}^{0\mu} = 0$, $\bar{\theta}_{\mu\nu}$ is a very small constant anti-symmetric matrix element, it reflects the non-commutativity of the momenta, and $\bar{\theta}_{ij} = \epsilon_{ij}\bar{\theta}$. The $\tilde{\hbar}$ in Eq.(4) is the deformation of Planck constant on NC phase space, it has the form

$$\tilde{\hbar} = \hbar + \frac{\theta\bar{\theta}}{4\hbar}, \quad (5)$$

and hence the momentum operator on NC phase space can be written as

$$\hat{p}_\mu = -i\tilde{\hbar}\frac{\partial}{\partial x^\mu}. \quad (6)$$

From the above relations, we can obtain a generalized Bopp's shift as

$$\hat{x}_\mu = x_\mu - \frac{1}{2\hbar}\theta_{\mu\nu}p^\nu, \quad (7)$$

$$\hat{p}_\mu = p_\mu + \frac{1}{2\hbar}\bar{\theta}_{\mu\nu}x^\nu. \quad (8)$$

It is easy to check that the above generalized Bopp's shift is consistent with the algebraic relation (4). Then the star product on NC phase space can be easily defined as

$$\begin{aligned} (f * g)(x) &= f(x, p) e^{\frac{i}{2}\bar{\partial}_\mu^\mu \theta^{\mu\nu} \bar{\partial}_\nu^\nu + \frac{i}{2}\bar{\partial}_\mu^\mu \bar{\theta}^{\mu\nu} \bar{\partial}_\nu^\nu} g(x, p) \\ &= f(x, p) g(x, p) + \frac{i}{2}\theta^{\mu\nu} \partial_\mu^x f(x, p) \partial_\nu^x g(x, p) \\ &\quad + \frac{i}{2}\bar{\theta}^{\mu\nu} \partial_\mu^p f(x, p) \partial_\nu^p g(x, p) + \mathcal{O}(\theta^2). \end{aligned} \quad (9)$$

In NC quantum mechanics and NC quantum field theory, the star product between two fields on NC phase space can be replaced by the generalized Bopp's shift (7) for coordinates and (8) for momenta.

Now we would like to stress that our formulation above is well defined on 2-dimensional NC phase space, and this formulation can be generalized only to any even dimensional case.

III. LORENTZ TRANSFORMATION ON NON-COMMUTATIVE PHASE SPACE

Let's first discuss the Lorentz transformation on NC space-time[8]. In an analogous way as in commutative space-time, on NC space one could introduce a NC Lorentz transformation as $\hat{x}_\mu = \Lambda_\mu^\nu \hat{x}_\nu$. But this kind of definition of Lorentz transformation is not consistent with the algebra (1), since it would require $\theta_{\mu\nu}$ transforms as $\Lambda_\mu^\alpha \Lambda_\nu^\beta \theta_{\alpha\beta}$, this makes little sense, because $\theta_{\mu\nu}$ is a constant and does not change under Lorentz transformation.

From the Bopp's shift (3) on NC space one finds that

$$x_\mu = \hat{x}_\mu + \frac{1}{2\hbar}\theta_{\mu\nu}\hat{p}^\nu, \quad p_\mu = \hat{p}_\mu, \quad (10)$$

On commuting space-time we can define a Lorentz transformation as follows

$$x'_\mu = \Lambda_\mu^\nu x_\nu, \quad (11)$$

which leaves the interval

$$s^2 = \eta_{\mu\nu} x^\mu x^\nu \quad (12)$$

invariant if $\eta_{\mu\nu} \Lambda_\alpha^\mu \Lambda_\beta^\nu = \eta_{\alpha\beta}$. Under the Lorentz transformation (11), the momentum p_μ transforms as a Lorentz vector

$$p'_\mu = \Lambda_\mu^\nu p_\nu. \quad (13)$$

From the Bopp's shift (3), we can obtain the following Lorentz transformation on NC space-time which is induced by the Lorentz transformation (11) and (13)

$$\begin{aligned} \hat{x}'_\mu &= x'_\mu - \frac{1}{2\hbar}\theta_{\mu\nu}p'^\nu = \Lambda_\mu^\nu x_\nu - \frac{1}{2\hbar}\theta_{\mu\nu}\Lambda_\rho^\nu p^\rho \\ &= \Lambda_\mu^\nu \hat{x}_\nu + \frac{1}{2\hbar}\Lambda_\mu^\nu \theta_{\nu\rho} \hat{p}_\rho - \frac{1}{2\hbar}\theta_{\mu\nu} \Lambda_\rho^\nu \hat{p}^\rho. \end{aligned} \quad (14)$$

The above equation defines the non-commutative Lorentz transformation on NC space-time. From this transformation one may note that rather than $\theta^{\mu\nu}$ transforms as a Lorentz tensor, the $\theta^{\mu\nu}p_\nu$ transforms as a Lorentz vector. So it is easy to check that the commutation relation (1) on NC space-time is invariant under this transformation. And, obviously, when $\theta^{\mu\nu} \rightarrow 0$, the NC Lorentz transformation above becomes usual Lorentz transformation on commuting space-time. From the shift (10) and the Lorentz invariant interval (12), one finds the square of the NC length

$$s_{nc}^2 = \hat{x}^\mu \hat{x}_\mu + \frac{1}{\hbar}\theta_{\mu\nu}\hat{x}^\mu \hat{p}^\nu + \frac{1}{4\hbar^2}\theta^{\mu\alpha}\theta_{\mu\beta}\hat{p}_\alpha \hat{p}^\beta. \quad (15)$$

Straight forward calculation shows that the s_{nc}^2 is invariant under NC Lorentz transformation (14). So we have defined a non-commutative Lorentz transformation which leaves s_{nc}^2 invariant.

Now we are in position to generalize the Lorentz transformation on NC space to NC phase space. From the generalized Bopp's shift (7) and (8) on NC phase space, we obtain it's inverse transformations

$$x_\mu = \gamma(\hat{x}_\mu + \frac{1}{2\hbar}\theta_{\mu\nu}\hat{p}^\nu), \quad (16)$$

$$p_\mu = \gamma(\hat{p}_\mu - \frac{1}{2\hbar}\bar{\theta}_{\mu\nu}\hat{x}^\nu), \quad (17)$$

where $\gamma = 4\hbar^2/(4\hbar^2 - \theta\bar{\theta})$. The Lorentz transformations (11) and (13) induce the following transformations on NC phase space

$$\hat{x}'_\mu = \Lambda_\mu^\nu x_\nu - \frac{1}{2\hbar}\theta_{\mu\nu}\Lambda_\rho^\nu p^\rho, \quad (18)$$

$$\hat{p}'_\mu = \Lambda_\mu^\nu p_\nu + \frac{1}{2\hbar}\bar{\theta}_{\mu\nu}\Lambda_\rho^\nu x^\rho. \quad (19)$$

Inserting Eqs.(16) and (17) into Eqs.(18) and (19), one obtains

$$\begin{aligned}\hat{x}'_\mu &= \gamma(\Lambda_\mu^\nu \hat{x}_\nu + \frac{1}{2\hbar} \Lambda_\nu^\mu \theta_{\nu\lambda} \hat{p}^\lambda - \frac{1}{2\hbar} \theta_{\mu\nu} \Lambda_\lambda^\nu \hat{p}^\lambda \\ &+ \frac{1}{4\hbar^2} \theta_{\mu\nu} \bar{\theta}^{\lambda\alpha} \Lambda_\lambda^\nu \hat{x}_\alpha),\end{aligned}\quad (20)$$

$$\begin{aligned}\hat{p}'_\mu &= \gamma(\Lambda_\mu^\nu \hat{p}_\nu - \frac{1}{2\hbar} \Lambda_\nu^\mu \bar{\theta}_{\nu\lambda} \hat{x}^\lambda - \frac{1}{2\hbar} \bar{\theta}_{\mu\nu} \Lambda_\lambda^\nu \hat{x}^\lambda \\ &- \frac{1}{4\hbar^2} \bar{\theta}_{\mu\nu} \theta^{\lambda\alpha} \Lambda_\lambda^\nu \hat{p}_\alpha).\end{aligned}\quad (21)$$

Eqs.(20) and (21) define the non-commutative Lorentz transformations on NC phase space, the $\theta_{\mu\nu} p^\nu$ and $\bar{\theta}_{\mu\nu} x^\nu$ transform as Lorentz vector on NC phase space. When $\bar{\theta} \rightarrow 0$, the Lorentz transformations (20) and (21) on NC phase space return to the Lorentz transformations on NC space. Using the inverse of the generalized Bopp's shift (16) and (17), one finds that the square of the non-commutative length on NC phase space is given by

$$s_{ncps}^2 = \gamma^2(\hat{x}^\mu \hat{x}_\mu + \frac{1}{\hbar} \theta_{\mu\nu} \hat{x}^\mu \hat{p}^\nu + \frac{1}{4\hbar^2} \theta^{\mu\alpha} \theta_{\mu\beta} \hat{p}_\alpha \hat{p}^\beta). \quad (22)$$

One can check that s_{ncps}^2 is left invariant by the NC Lorentz transformations on NC phase space.

It is straightforward to extend our results above to a Poincare transformation, since a shift by a constant of the non-commutative coordinates is compatible with the algebraic relations Eq.(1) on NC space and Eq.(4) on NC phase space. An infinitesimal non-commutative Poincare transformation $\Lambda_\nu^\mu = \delta_\nu^\mu + \omega_\nu^\mu$, $a^\mu = \epsilon^\mu$ is implemented by the operator

$$U(1 + \omega, \epsilon) = 1 + \frac{i}{2} \omega_{\mu\nu} J^{\mu\nu} - i \epsilon_\mu p^\mu + \dots \quad (23)$$

with $J_{\mu\nu} = x_\mu p_\nu - x_\nu p_\mu$. The operator is un-deformed, because the NC Poincare transformation is induced by the Poincare transformation on commuting space. So the Lie algebra of the Lorentz group is also un-deformed,

$$\begin{aligned}[J_{\mu\nu}, J_{\rho\sigma}] &= -i(\eta_{\nu\rho} J_{\mu\sigma} - \eta_{\mu\rho} J_{\nu\sigma} - \eta_{\sigma\mu} J_{\rho\nu} + \eta_{\sigma\nu} J_{\rho\mu}), \\ [p_\mu, J_{\rho\sigma}] &= -i(\eta_{\mu\rho} p_\sigma - \eta_{\mu\sigma} p_\rho), \\ [p_\mu, p_\nu] &= 0.\end{aligned}\quad (24)$$

One may apply our formulation to field theory. To do so, derivatives have to be defined, by Ref.[14], the derivative on NC space can be defined as

$$\begin{aligned}\hat{\partial}_\mu^x f(x, p) &= -i\theta_{\mu\nu}^{-1}[\hat{x}^\nu, f(x, p)] \\ &= -i\theta_{\mu\nu}^{-1}[x^\nu - \frac{1}{2\hbar} \theta^{\nu\alpha} p_\alpha, f(x, p)].\end{aligned}\quad (25)$$

Similarly, on NC phase space we define the derivative of momentum as follows

$$\begin{aligned}\hat{\partial}_\mu^p f(x, p) &= -i\bar{\theta}_{\mu\nu}^{-1}[\hat{p}^\nu, f(x, p)] \\ &= -i\bar{\theta}_{\mu\nu}^{-1}[p^\nu + \frac{1}{2\hbar} \bar{\theta}^{\nu\alpha} x_\alpha, f(x, p)].\end{aligned}\quad (26)$$

Under NC Lorentz transformations, the derivatives transform as

$$\begin{aligned}\hat{\partial}_\mu^{'x} &= \theta_{\mu\alpha}^{-1} \Lambda_\beta^\alpha \theta^{\beta\sigma} \hat{\partial}_\sigma^x, \\ \hat{\partial}_\mu^{'p} &= \bar{\theta}_{\mu\alpha}^{-1} \Lambda_\beta^\alpha \bar{\theta}^{\beta\sigma} \hat{\partial}_\sigma^p.\end{aligned}\quad (27)$$

Now let's consider a non-commutative action for a Dirac fermion coupled to a Yang-Mills gauge field, the action, in 2 + 1 dimension, is given by

$$S = \int d^3\hat{\Psi}(\hat{x})(i\hat{\mathcal{D}} - m)\hat{\Psi}(\hat{x}) - \frac{1}{2} \int d^3x \text{Tr} \hat{F}_{\mu\nu}(\hat{x}) \hat{F}^{\mu\nu}(\hat{x}). \quad (28)$$

Under the NC Lorentz transformations, Ref.[8] found that the NC Yang-Mills potential transforms as

$$\hat{A}_\mu' = \theta_{\mu\alpha}^{-1} \Lambda_\beta^\alpha \theta^{\beta\sigma} \hat{A}_\sigma, \quad (29)$$

the NC covariant derivative transforms as

$$\hat{D}_\mu' = \theta_{\mu\alpha}^{-1} \Lambda_\sigma^\alpha \theta^{\sigma\alpha} \hat{D}_\sigma, \quad (30)$$

and the field strength tensor transforms as

$$\hat{F}_{\mu\nu}' = \theta_{\mu\rho}^{-1} \Lambda_\sigma^\rho \theta^{\sigma\alpha} \theta_{\nu\zeta}^{-1} \Lambda_\eta^\zeta \theta^{\eta\beta} \hat{F}_{\alpha\beta}. \quad (31)$$

as well as the NC spinor field transforms as

$$\hat{\Psi}' = \exp(-\frac{i}{2} \omega^{\alpha\beta} S_{\alpha\beta}) \hat{\Psi}, \quad (32)$$

with $S_{\mu\nu} = \frac{i}{4}[\gamma_\mu, \gamma_\nu]$. If the fields are taken in the enveloping algebra, the classical field, namely, the leading order of Seiberg-Witten map, also transforms according to (29)-(32). Though the equations (29)-(32) have the same form both for 2 + 1 dimensional NC space time and the 3 + 1 dimensional NC space time in [8], because of the singularity of the matrix $\Theta = (\theta_{ij})$ in 3 + 1 dimensional NC space-time, the equations (29)-(32) can not be applicable to 3 + 1 dimensional NC space time, detailed discussions will be given in the conclusion part.

To replace the noncommutative argument of the function by a commutative one, we need to introduce a star product, $f(\hat{x})g(\hat{x}) = f(x) \star g(x)$, the star product here is given in Eq.(2) for NC space and defined in Eq.(9) for NC phase space. It is easy to verify that these star product is invariant under NC Lorentz transformations. After replacing NC coordinates by the commutative ones through star product and expanding the fields in the enveloping algebra by using Seiberg-Witten Map, the Eq.(28), to the first order of θ , reads

$$\begin{aligned}S &= \int d^3x \left[\bar{\psi}(i\mathcal{D} - m)\psi - \frac{1}{4} \theta^{\mu\nu} \bar{\psi} F_{\mu\nu} (i\mathcal{D} - m)\psi \right. \\ &- \frac{1}{2} \theta^{\mu\nu} \bar{\psi} \gamma^\rho F_{\rho\mu} iD_\nu \psi - \frac{1}{2} \text{Tr} F_{\mu\nu} F^{\mu\nu} \\ &+ \left. \frac{1}{4} \theta^{\mu\nu} \text{Tr} F_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} - \theta^{\mu\nu} \text{Tr} F_{\mu\rho} F_{\nu\sigma} F^{\rho\sigma} \right] + \mathcal{O}(\theta^2),\end{aligned}\quad (33)$$

which is invariant under noncommutative Lorentz transformation.

IV. CONCLUSION REMARKS

In this paper, we give a description of NC phase space, where the star product can be easily defined(see Eq.(9)), the Planck constant is modified on NC phase space(see Eq.(5)).

Following the Ref.[8], we define a non-commutative Lorentz transformations both on NC space and NC phase space in $2 + 1$ dimensions. The basic idea is to define the Lorentz transformations for commutative coordinate (11) and momentum (13) then to feed back these transformation to the non-commutative sectors via variables transformations (16) and (17). The algebraic relations both on NC space and NC phase space are invariant under NC Lorentz transformations, and the θ -expanded gauge field action is also invariant under NC Lorentz transformations.

In this paper though the Greek indices are introduced, we assume μ, ν run as i, j , take values from 1 to 2, because we set $\theta_{0\mu} = 0$. Why we consider a 2-dimensional NC space and NC phase space instead of 3-dimensional case. The main reason is that on 3-dimensional NC space and NC phase space the non-commutative parameters θ_{ij} and $\bar{\theta}_{ij}$ can be considered as anti-symmetric matrices, $\Theta = (\theta_{ij})$, $\bar{\Theta} = (\bar{\theta}_{ij})$, the elements can be written as

$$\theta_{ij} = \epsilon_{ijk}\theta_k, \quad \bar{\theta}_{ij} = \epsilon_{ijk}\bar{\theta}_k. \quad (34)$$

i.e.

$$\Theta = \begin{pmatrix} 0 & \theta_3 & -\theta_2 \\ -\theta_3 & 0 & \theta_1 \\ \theta_2 & -\theta_1 & 0 \end{pmatrix}, \quad \bar{\Theta} = \begin{pmatrix} 0 & \bar{\theta}_3 & -\bar{\theta}_2 \\ -\bar{\theta}_3 & 0 & \bar{\theta}_1 \\ \bar{\theta}_2 & -\bar{\theta}_1 & 0 \end{pmatrix}. \quad (35)$$

It is easy to find that these two anti-symmetric matrices are singular matrices, because $\det\Theta = \det\bar{\Theta} = 0$. For these reason we can not define the inverse of these matrices, and terms related to θ_{ij}^{-1} or $\bar{\theta}_{ij}^{-1}$ will make no sense on 3-dimensional NC space and NC phase space. This problem also exists in Ref.[8], for example, the Eqs.[17-21] of the Ref.[8] make little sense in $3 + 1$ dimensions. The NC phase space formulation and the NC Lorentz transformation of this paper can be easily extended to any even dimensional space, however, for any odd dimensional case, the anti-symmetric matrices Θ and $\bar{\Theta}$ are singular, the method here would not be applicable, and we will discuss it in our forthcoming studies.

Acknowledgments

Kang Li would like to thank Prof. S. Randjbar-Daemi for his kind invitation and warm hospitality during his visit at the ICTP. This work is supported in part by the National Natural Science Foundation of China (10575026 and 10465004, 10665001). The authors also grateful to the support from the Abdus Salam ICTP, Trieste, Italy.

-
- [1] N. Seiberg and E. Witten, JHEP **032**, 9909(1999), hep-th/9908142.
 - [2] M. Chaichian, M. M. Sheikh-Jabbari and A. Tureanu, Phys. Rev. Lett. **86**, 2716(2001); M. Chaichian, A. Demichev, P. Presnajder, M. M. Sheikh-Jabbari and A. Tureanu, Nucl. Phys. B **611**, 383(2001), hep-th/0101209.
 - [3] M. Chaichian, P. Presnajder, M. M. Sheikh-Jabbari and A. Tureanu, Phys. Lett. B **527**, 149 (2002) 149-154; H. Falomir, J. Gamboa, M. Loewe, F. Mendez and J. C. Rojas, Phys. Rev. D **66**, 045018(2002), hep-th/0203260; Omer F. Dayi and Ahmed Jellal, J.Math.Phys. **43**, 4592 (2002), hep-th/0111267.
 - [4] Kang Li and Sayipjamal Dulat, Eur. Phys. J. C **46**, 825 (2006); Kang Li and Jianhua Wang, Eur. Phys. J. C **50**, (2007)1007; J. Phys. A: Math Theor. **40** (2007)2197.
 - [5] B.Mirza and M.Zarei, Eur. Phys. J. C **32** (2004)583; B.Mirza, R.Narimani and M.Zarei, Eur. Phys. J. C **48** (2006)641.
 - [6] Kang Li, Jianhua Wang and Chiyi Chen, Mod. Phys. Lett. A **20**, 2165-2174 (2005).
 - [7] Michal Demetrian and Denis Kochan, hep-th/0102050; Jian-zu Zhang, Phys. Lett. B **584**, 204(2004).
 - [8] X. Calmet, Phys.Rev. D **71** (2005) 085012.
 - [9] J. Wess, hep-th/0408080; M. Chaichian, P. P. Kulish, K. Nishijima, and A. Tureanu, Phys. Lett. B **604**, 98(2004).
 - [10] J. Lukierski, A. Nowicki, and H. Ruegg, Phys. Lett. B **293**, 344(1992).
 - [11] J. Gomis, T. Mehen, Nucl. Phys.B **591**, 265(2000).
 - [12] L. Alvarez Gaume, J. L. F. Barbon, R. Zwicky, JHEP **05**, 057(2001).
 - [13] N. Seiberg, L. Susskind, N. Toumbas, JHEP **06**,044(2000).
 - [14] L.Alvarez-Gaume and Spenta R. Wadia, Phys. Lett. **B501** (2001)319; S-C. Chu, Nucl. Phys. B **550**, 151 (1999).